Improved Solution for Traffic Model with a Rectangular Service Zone

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Abstract: *Fluid traffic operation has always been considered an essential condition when assessing the managerial performance of a traffic net design. Deciding the length and width of a bus service route and headways are also important issues because they directly influence the service level for passengers and profit for operators. Thus, many scholars focus their attention on finding the optimal solution for traffic models. This research examined three published papers and aimed to revise their results for a bus model with a rectangular service area and geometric combination of costs and profits. Questionable results are first pointed out and improvements are provided to prove the existence and uniqueness of the optimal solution. Nonetheless, the relation between headway and route width was verified to still hold true. The same numerical example as used in the reviewed papers is illustrated to show that the proposed derivation dramatically improves profit by 135.24%.*

Keywords: *Traffic model; Bus service zones; Optimal solution.*

1. Introduction

Traffic models with formulated optimal solutions are an appealing research topic since the relations among parameters and variables indicate their respective importance. Traffic models can be divided into two cases: (a) complicated models with many variables and constraints such that researchers cannot decide the optimal (or near-optimal) solution, and (b) deterministic models with two or three variables and a few constraints such that practitioners tried to solve the optimal solution. The majority of traffic modes can be categorized as complicated models. we just name a few to illustrate them. Cipriani et al. [1] constructed a bus network design to find routes and frequency by genetic methods and heuristic route-generation algorithms. Gallo et al. [2] studied frequency issue for bus route in transportation with demand elasticity among different sectors: rail, bus, and private car and applied the heuristic local search algorithm, scatter search, and genetic algorithm to determine the frequency. Asadi Bagloee and Ceder [3] developed a method to filter nodes and group them to contender stops, and then they decided transit routes by genetic methods. For transit network design problems, Mauttone and Urquhart [4] applied insertion algorithms to construct a bus routes system. Guihaire and Hao [5] examined sixty-nine papers to present a comprehensive review of transit system design and scheduling problems that was classified by the solution approach and the problem tackled. Roca-Riu et al. [6] formulated the interurban bus system in city centers to adopt the Tabu search to find the balance between operator cost and user cost. According to Guihaire and Hao [5], the solution approaches for traffic models can be categorized into four parts: (1) evolutional approach, (2) heuristic method, (3) neighborhood search, and (4) hybrid methods. This paper was developed based on a series of papers: Chang and Schonfeld [7], Chang and Schonfeld [8], Imam [9], Yang et al. [10], Tung et al. [11], and Chen and Julian [12] that are classified as deterministic traffic models which are not belonged to the above mentioned four parts proposed by Guihaire and Hao [5]. Initiated from Chang and Schonfeld [7], our series of papers are dedicated to derive the exact minimum cost (or maximum profit) model and then find the optimal solution. On the other hand, our proposed exact method was not included in the preceding four parts to show that the majority research papers are focused to consider a real word problem to describe the situation as detailed as possible such that many variables and constraints are contained in their traffic systems and then their traffic systems become too complicate beyond their ability to control. Consequently, the majority research papers cannot find their optimal solution. However, we can claim that our exact approach to derive the optimal solution for traffic models to reveal relations among variables under the optimal environment. Our transit network model will obtain the optimal solution by the analytic method. Some researcher may consider that our transit system is a relatively over-simplified model, however, our optimal solution through analytical approach can offer important relations among variables and given parameters. Based on these relations, practitioners can decide the relative importance of those given parameters. Consequently, they can collect real data to decide the estimated parameters under the budget restrictions, and furthermore to develop new traffic systems to mimic the real word phenomenon. Motivated by Cipriani et al. [1], Asadi Bagloee and Ceder [3], Mauttone and Urquhart [4], Guihaire and Hao [5], and Roca-Riu et al. [6], we claim the direction for future research will integrate the following into their transit network model: (1) bus operating times, (2) synchronization for adjacent transfers among several bus lines, (3) bus capacities, (4) bus fleet size, (5) the transfer phenomena with up to two transfers, and (6) bus departure schedules to develop more realistic transit models. Chuang [13] examined a detailed study for Cipriani et al. [1] to point out several doubtful findings: (i) Definition of LTR, (ii) Restriction of routes, (iii) Crossover for two routes, (iv) Computation in Table 4, (v) An increase of the transit demand of about 2.5%, and (vi) 50% should be revised to 60%, of Cipriani et al. [1] to help researchers understand Cipriani et al. [1]. Based on Chuang [13], we can say that the transportation system of urban area of Rome is too complicated to handle by Cipriani et al. [1]. Chao [14] provided a comprehensive examination for Mauttone and Urquhart [4] for bus route development to decide the transit system in Rivera, Uruguay. Chao [14] showed that several dubious findings in Mauttone and Urquhart [4]: (a) Execution time, (b) How many bus routes in Rivera, Uruguay? (c) Diversity measure, (d) Their passengers demand matrix is not suitable to derive frequencies, and then Chao [14] offered improvements for these questionable results to assist researchers to construct further transit network in the future. Chu and Lin [15] demonstrated a detailed consideration for Roca-Riu et al. [6]. Several suspicious findings were shown by Chu and Lin [15]: (a) The meaning of α, (b) The meaning of two lower bounds, (c) α=1 and δ=0.2 in iterations, (d) The neighborhood size, (e) Citation from Table 1, (f) The range of saving, and (g) The title. we can claim that the traffic model of Barcelona proposed by Roca-Riu et al. [6] is too tedious to beyond their ability to deal with it. Chu and Hopscotch [16] run a thoughtful review of Asadi Bagloee and Ceder [3] to illustrate that (a) Zone or zonal level is not defined, (b) Simplify maximum to minimum, (c) A typo in Eq. (3), (d) Normalization of demand, (e) Balancing factor for demand, (f) A typo in Eq. (5), (g) ϱ should revise from 0≤ϱ≤1 to (1⁄21)≤ϱ≤1 (h) Normalization of attraction index, (i) How to derive i_3, and (j) cannot converge to zero. The transit system of Chicago proposed by Asadi Bagloee and Ceder [3] contained questionable derivations to indicate that is not easy to solve a real word problem under many variables and constraints. After we review Chuang [13], Chao [14], Chu and Lin [15] and Chu and Hopscotch [16] to reveal that those transit models tried to describe the real word traffic problems but failed to offer their optimal solutions, we came back to the research direction with compact and simplified transit models. There are several papers that had worked on formulated optimal solutions of traffic models. The following contains a brief review of selected, closely-related papers to emphasize the trend of literature development: Kocur and Hendrickson [17] developed a traffic model to design a local bus service with demand equilibrium with three objective functions: (a) maximize profit or minimize deficit, (b) maximize the sum of operator profit and a fraction of net user benefit, (c) maximize net user benefit subject to a deficit constraint. For the profit-maximization case, where the variables are bus route width, headway and fare, a quadratic polynomial for bus route width was derived. The highest term of the spacing between parallel bus routes was neglected in order to derive an approximated solution. However, it is already known that there exists a formulated solution for a quartic polynomial such that all variables including bus route width, headway and fare have their own optimal solutions. For the profit-maximization case with a vehicle capacity constraint it resulted in a cubic polynomial for bus route width. The highest term was again neglected in order to derive an approximated solution. Therefore, it is safe to say that it is possible to find formulated optimal solutions from the general findings of these relevant researches. Chang and Schonfeld [7] studied traffic models for a rectangular service area with four different demand restrictions: case 1, steady fixed demand; case 2, cyclical fixed demand; case 3, steady equilibrium demand; and case 4, cyclical equilibrium demand. For cases 1 and 2, Chang and Schonfeld [7] obtained an optimal solution for headway and service route width. For cases 3 and 4, a quartic polynomial was derived for service route width and the highest term was neglected for the same reason as discussed above. Chang and Schonfeld [8] generalized Chang and Schonfeld [7] in order to include the service route length as a new variable. Chang and Schonfeld [8] obtained optimal solutions for service route length, headway and service route width. Imam [9] constructed a traffic model similar to Chang and Schonfeld [8] that is a generalization from linear combination to exponential expressions and found solutions for the same variables. Chen and Julian [12] pointed out the solution contains questionable results and then proposed a revision for Imam [9]. Tirachini et al. [18] developed a traffic model with elastic demand to maximize the profit where the variables are frequency and fare. This previous study claimed that it is not possible to obtain an analytical expression for optimal frequency. However, after investigation, the optimal frequency derived is a root of a cubic polynomial such that the solution is possible for their optimal frequency. Consequently, the optimal fare also has a formulated solution. Li et al. [19] constructed a maximization traffic model for a rail transit line where the variables are rail line length, station locations, headway and fare. Solutions were derived for the latter two variables but not for rail line length and station locations. Yang et al. [10] pointed out a critical issue in the result of Chang and Schonfeld [8] wherein a negative sign was misplaced on a positive term and thus, presented a revision to amend such a questionable result. Moreover, Yang et al. [10] constructed an iterative algorithm to find an alternative sequence that converges to the optimal service route length. Recently, Tung et al. [11] further improved the derivation of Yang et al. [10] and illustrated that there are three different cases whereas Yang et al. [10] only considered one. Tung et al. [11] pointed out that it is misleading to consider the results of Chang and Schonfeld [8] and Yang et al. [10] as real solutions. Subsequently, Tung et al. [11] examined the positive solution for the service route length. On the other hand, Amiripour et al. [20] criticized traffic models with rectangular service zones and mentioned that it is not appropriate to partition a city into triangular bus service zones as executed by Chang and Schonfeld [7]. However, further reading into Chang and Schonfeld [7] proved that the division method could be used to divide any region into rectangular bus service zones. The key issue for this article was to discuss one rectangular service zone with the variables: zone length, zone width and multiple headways.

The following sections are organized as follows: section 2 presents the notation and assumptions used; section 3 examines the traffic model with rectangular service area proposed by Imam [9]; section 4 discusses the solution procedure of Chen and Julian [12] and then we point out their questionable findings; section 5 focuses on our proposed solution approach to prove the existence and uniqueness of the optimal solution; section 6 sets out the numerical example where the data was proposed by Kocur and Hendrickson [17]; and section 7 presents the main findings, conclusion, and recommendations for further research.

2. Notation and Assumptions

To be compatible with Imam [9] and Chen and Julian [12], the same parameters and variables are used and we list them in the following table 1. Parameter values are cited from Kocur and Hendrickson [17] as Chen and Julian [12].

Table 1: Definitions of parameters and variables.

Notation	Definition	Parameter values
a_1	= model choice coefficient: transit constant	0.38
a_2	= model choice coefficient: wait and walk time	– 0.0081
a_3	= model choice coefficient: in-vehicle travel time	– 0.0033
a_4	= model choice coefficient: fare	– 0.0014
a_5	= model choice coefficient: auto time and cost	0.0328
b	= spacing between bus stops along routes (mile)	0.2
c	= bus operating cost (dollar/hour)	0.5
d	= average passenger trip length (mile/hour)	3
F	= fare for bus service (dollars)	0.74
h (variable)	= headway on bus route (minute)	
j	= average walking speed (mile/min);	0.05
k	= ratio of expected user wait time-to-headway	0.4
L (variable)	= length of analysis area (mile)	
p	= trip density by all other modes (trip/mile^2/min)	3.59
q	= vehicle capacity (passenger/bus)	45
s (variable)	= spacing between parallel bus routes (mile)	
T	= time period of analysis (min)	60
v	= average bus speed, including stop (mile/min)	0.167
W	= width of analysis or service area (mile)	4

This traffic model with steady fixed demand is designed for a rectangular service zone with three variables: length L, width s, and headway h. In Chang and Schonfeld [7], there are only two variables: zone width s and headway h. Chang and Schonfeld [8] generalized traffic models to treat zone length, L, as a variable. Imam considered a bus service model with a rectangular service zone which has a fixed zone width and the zone length treated as a decision variable. The service zone is partitioned evenly into N bus routes so one bus route width is $s = W/N$. The access distance for a passenger is evaluated as $(s+b)/4$ with the average walking speed j such that $(s+b)/4j$ is the access time. Waiting time is derived as hk with ratio of expected user wait time to headway k. The in-vehicle time is assumed as d/v with the average passenger trip length d and average bus speed

v. The total fare for the rectangular service area is $TpLWF$ with the time period of analysis, T, where p is trip density, with the length, L; the width, W; and the fare, F. The bus operating cost is $2L(W/s)Tc/hv$ with the round trip length, $2L$; the number of bus route, W/s; time period of analysis, T; the bus operating cost per hour, c; the headway, h and average bus speed, v.

To simplify the expressions, we assume that

$$B_1 = pa_1(d/v)^{a_3} F^{a_4} d^{a_5}, \quad (1)$$

$$B_2 = TWFB_1, \quad (2)$$

$$B_3 = 2WTc/v, \quad (3)$$

and

$$B_4(h,s) = kh + ((s+b)/4j). \quad (4)$$

3. Review of Imam [9]

There are 14 articles that cited Imam [9] in their references. Six of them: Tom and Mohan [21], Agrawal and Mathew [22], Yepes and Medina [23], Kepaptsoglou and Karlaftis [24], Ranjbari et al. [25], and Xiong et al. [26], only mentioned it in their introduction without any discussion for the traffic model or its solution procedure. Yang et al. [10] studied the rectangular bus service zone of Chang and Schonfeld [8] and improved the algebraic derivation. An analytical approach was also developed to construct an alternating series that will iteratively converge to the desired route length. Hung and Julianne [27] used the bisection method to locate the optimal service length for the bus model of Chang and Schonfeld [8]. Kim and Schonfeld [28] mentioned that Imam [9] extended Kocur and Hendrickson [17] for a maximum profit model from a linear relation to a geometric relation. Imam [9] did not provide numerical examples for the bus model. Hence, the best alternative is to refer to the numerical examples of Kocur and Hendrickson [17] in order to check the proposed procedure of Imam [9]. Tung et al. [11] further improved Yang et al. [10] to show that Yang et al. [10] should not consider the uniqueness of the real optimal solution. Instead, Tung et al. [11] developed three cases, which all have their own unique positive solution. Yang et al. [29] studied the first bus model of Imam [9] to show that the solution provided by Imam [9] is questionable because it will yield a negative value for the route width. Moreover, Chen and Julian [12] pointed out that the cubic polynomial of s proposed by Imam [9] is questionable since "*A*" was treated as a constant, but "*A*" contains the variable s. Luo [30] showed that in Tung et al. [11], they applied the Newton's method under a restriction $L \geq 1$ that is not satisfied by a real case of Keelung, Taiwan and then constructed a new auxiliary function to obtain a new starting point for the Newton's method. Chen and Julian [12] examined the second bus model of Imam [9] to show that his proposed solution is questionable and derived a new formulated optimal solution for headway.

In Chang and Schonfeld [8], Yang et al. [10] and Tung et al. [11], costs are linearly related with addition. In Imam [9], Yang et al. [19], and Chen and Julian [12], profits are exponentially related with multiplication. Hence, there is some similarity between the traffic models of Chang and Schonfeld [8] and Imam [9] except for the different solution structures. This paper shows that the solution structure of Imam [9] is more complicated than that of Chang and Schonfeld [8]. Based on the above discussion, it is safe to claim that all previous referred articles were not able to provide a careful examination for Imam [9]. Therefore, this paper focused on examining the second traffic model of Imam [9] with a capacity constraint and on proving the existence and uniqueness of the service route width. However, the formulated optimized solution of zone width is no longer reachable. Fortunately, the well-known relation between headway and service route width still is valid. Thus, it is possible to derive a formulated solution for the optimal service zone length with respect to the optimal service zone width and headway. The findings of this paper provide useful information for the relation between the optimal headway and the optimal service route width.

4. Review of Chen and Julian [12]

The traffic model examined by Chen and Julian [12] was based on Imam [9] and this paper follows Chen and Julian [12] with similar objective functions. Interested readers may refer to Imam [9] for the original derivations of the first and second models.

We use the following abbreviations $B_1 = pa_1(d/v)^{a_3} F^{a_4} d^{a_5}$, $B_2 = TWFB_1$, $B_3 = 2WTc/v$ and $B_4(h,s) = kh + ((s+b)/4j)$ to simplify expressions. The objective function, maximum Profit (mP) as proposed by Imam [9] in his first traffic model without capacity constraint, can be simplified as

$$mP(h,s,L) = \left(B_2 (B_4(h,s))^{a_2} - \frac{B_3}{hs} \right) L . \tag{5}$$

Yang et al. [19] pointed out that this model is unreasonable because if L is continuously extended, the profit will reach infinity. Thus, it revealed one of the severe problems of the first model in Imam [9].

On the other hand, the maximum profit problem with the capacity constraint, as the second traffic model proposed by Imam [9], is simplified as

$$mP = \left(B_2 (B_4(h,s))^{a_2} - \frac{B_3}{hs} \right) L , \tag{6}$$

with $B_1 hsL(B_4(h,s))^{a_2} \le q$. Chen and Julian [12] mentioned that the objective function is expressed as L multiplying a positive term so L should be as large as possible to imply that

$$B_1 hsL\left(kh + \frac{s}{4j} + \frac{b}{4j} \right)^{a_2} = q . \tag{7}$$

Based on Equation (7), Chen and Julian [12] further simplified the objective function of Equation (6) as

$$mP(h,s) = \left(\frac{B_2}{hs} - \frac{B_3}{hs\left(kh + \frac{s}{4j} + \frac{b}{4j} \right)^{a_2}} \right) \frac{q}{B_1} . \tag{8}$$

However, this paper proposes that the corrected expression should be derived as

$$mP(h,s) = \left(\frac{B_2}{hs} - \frac{B_3}{h^2 s^2\left(kh + \frac{s}{4j} + \frac{b}{4j} \right)^{a_2}} \right) \frac{q}{B_1} . \tag{9}$$

Equation (9) forms the first issue found in Chen and Julian [12].

Based on Equation (8), Chen and Julian [12] computed that $\frac{\partial}{\partial h} mP(h,s) = 0$ and $\frac{\partial}{\partial s} mP(h,s) = 0$ to imply

$$\frac{B_2}{h^2 s} = \frac{B_3}{h^2 s^2 (B_4(h,s))^{1+a_2}} \left[s B_4(h,s) + k a_2 hs \right] , \tag{10}$$

and

$$\frac{B_2}{hs^2} = \frac{B_3}{h^2 s^2 \left(B_4(h,s)\right)^{1+a_2}} \left[hB_4(h,s) + \frac{a_2}{4j} hs \right]. \tag{11}$$

Chen and Julian [12] rewrote Equations (10) and (11) as follows,

$$\frac{B_2}{4jh^2 s} = \frac{B_3}{h^2 s^2 \left(B_4(h,s)\right)^{1+a_2}} \left[\frac{s\,B_4(h,s)}{4j} + \frac{k\,a_2}{4j} hs \right], \tag{12}$$

and

$$\frac{kB_2}{hs^2} = \frac{B_3}{h^2 s^2 \left(B_4(h,s)\right)^{1+a_2}} \left[hk\,B_4(h,s) + \frac{ka_2}{4j} hs \right] = 0, \tag{13}$$

and then took the difference of Equations (12) and (13) to derive that

$$\frac{B_2}{h^2 s^2}\left(\frac{s}{4j} - hk \right) = \frac{B_3 B_4(h,s)}{h^2 s^2 \left(B_4(h,s)\right)^{1+a_2}} \left[\frac{s}{4j} - hk \right]. \tag{14}$$

Based on Equation (14), Chen and Julian [12] obtained two cases: Case A: $B_2 \left(B_4(h,s)\right)^{a_2} = B_3$, and Case B: $s = 4jkh$.

However, the appropriate way to merge Equations (10) and (11) into one compact relation, Equation (17) is as follows:

$$\frac{B_2}{hs} = \frac{B_3}{h^2 s^2 \left(B_4(h,s)\right)^{1+a_2}} \left[h\,s\,B_4(h,s) + h\,k\,a_2\,hs \right], \tag{15}$$

and

$$\frac{B_2}{hs} = \frac{B_3}{h^2 s^2 \left(B_4(h,s)\right)^{1+a_2}} \left[hsB_4(h,s) + \frac{s\,a_2}{4j} hs \right], \tag{16}$$

then take the difference of Equations (15) and (16) to derive that

$$s = 4jkh. \tag{17}$$

Hence, Case A mentioned above is deemed unnecessary, thus forming the second issue for Chen and Julian [12].

For Case B, Chen and Julian [12] plugged $s = 4jkh$ into Equation (8) to obtain that

$$mP(h) = \frac{q}{4jkB_1}\left(\frac{B_2}{h^2} - \frac{B_3}{h^2 \left(2kh + (b/4j)\right)} \right). \tag{18}$$

However, it must be pointed out that the correct substitution should be

$$mP(h) = \frac{q}{4jkB_1}\left(\frac{B_2}{h^2} - \frac{B_3}{h^2 \left(2kh + (b/4j)\right)^{a_1}} \right). \tag{19}$$

Hence, the objective function is proven to be erroneous and forms the third critical issue for Chen and Julian [12]. The following section shows our proposed solution procedure.

5. Our solution procedure

Our goal is to maximize

$$mP(h,s)=\left(\frac{B_2}{hs}-\frac{B_3}{h^2 s^2\left(kh+\frac{s}{4j}+\frac{b}{4j}\right)^{a_2}}\right)\frac{q}{B_1}. \quad (20)$$

$\frac{\partial}{\partial h}mP(h,s)=0$ and $\frac{\partial}{\partial s}mP(h,s)=0$ derive

$$\frac{B_2}{h^2 s}=\frac{B_3}{h^3 s^3 (B_4(h,s))^{1+a_2}}\left[2s\,B_4(h,s)+k\,a_2\,hs\right], \quad (21)$$

and

$$\frac{B_2}{hs^2}=\frac{B_3}{h^3 s^3 (B_4(h,s))^{1+a_2}}\left[2hB_4(h,s)+\frac{a_2}{4j}hs\right]. \quad (22)$$

Equations (21) and (22) derive

$$\frac{B_2}{B_3}h^2 s^2 (B_4(h,s))^{1+a_2}=h(2sB_4(h,s)+ka_2hs)=s\left(2hB_4(h,s)+\frac{a_2}{4j}hs\right), \quad (23)$$

and thus,

$$kh=\frac{s}{4j}. \quad (24)$$

Using Equation (24), the objective function can be further simplified to

$$mP(s)=\frac{4jkq}{B_1}\left(\frac{B_2}{s^2}-\frac{4jk(4j)^{a_2}B_3}{s^4(2s+b)^{a_2}}\right). \quad (25)$$

Subsequent derivation yields

$$\frac{d}{ds}mP(s)=\frac{8jkq}{B_1 s^5 (2s+b)^{1+a_2}}f(s) \quad (26)$$

, where

$$f(s)=4jk(4j)^{a_2}B_3(2b+(4+a_2)s)-B_2 s^2(2s+b)^{1+a_2}. \quad (27)$$

Analyzing $f(s)$ obtains

$$f'(s)=4jk(4j)^{a_2}B_3(4+a_2)-2B_2 s(2s+b)^{1+a_2}-2(1+a_2)B_2 s^2(2s+b)^{a_2}, \quad (28)$$

and

$$f''(s)=-2B_2\left[(2s+b)^{1+a_2}+4s(1+a_2)(2s+b)^{a_2}+2a_2(1+a_2)s^2(2s+b)^{a_2-1}\right.. \quad (29)$$

From Equation (29), it is established that $f''(s)<0$ for $s>0$, $f'(s)$ is a decreasing function from $\lim_{s\to 0} f'(s)= 4jk(4j)^{a_2} B_3(4+a_2)>0$ to $\lim_{s\to\infty} f'(s)=-\infty$. Hence, there is a unique point, say $s^{\#}$ with $f'(s^{\#})=0$.

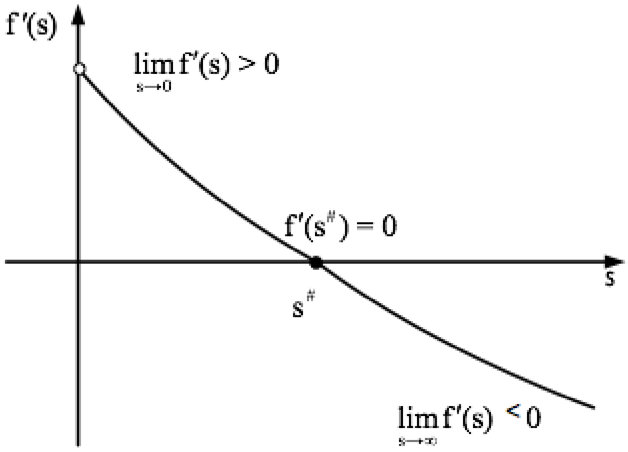

Figure 1. Graph of $f'(s)$.

From $f'(s)>0$ for $0<s<s^{\#}$ and $f'(s)<0$, for $s^{\#}<s<\infty$, we imply that $f(s)$ is an increasing function for $0<s<s^{\#}$ and a decreasing function for $s^{\#}<s<\infty$. Owing to $f(0)=8bjk(4j)^{a_2} B_3>0$ and $\lim_{s\to\infty} f(s)=-\infty$, there is a unique point, say s^{*} that satisfies $f(s^{*})=0$.

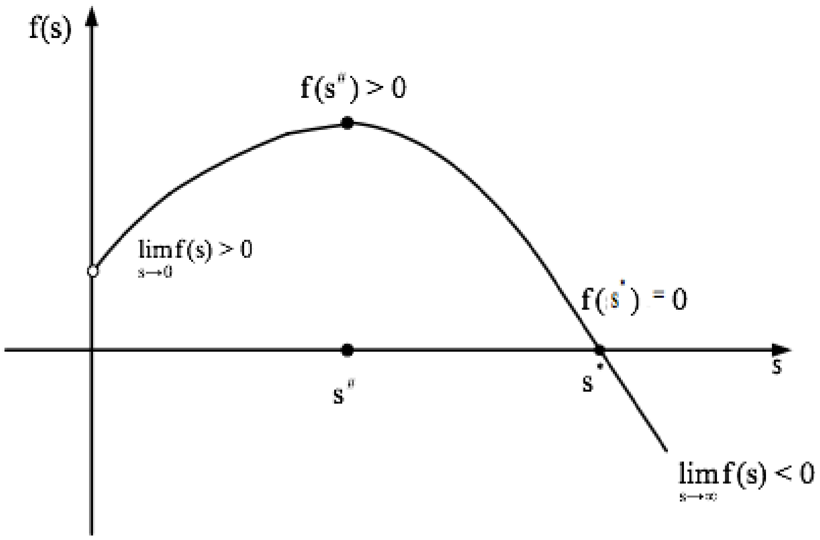

Figure 2. Graph of f(s).

Therefore, $f(s)>0$ for $0<s<s^{*}$ and $f(s)<0$, for $s^{*}<s<\infty$, the following are implied: $\frac{d}{ds}mP(s)>0$ for $0<s<s^{*}$ and $\frac{d}{ds}mP(s)<0$, for $s^{*}<s<\infty$.

Therefore, $mP(s)$ increases for $0 < s < s^*$ and decreases for $s^* < s < \infty$ to imply that s^* is the maximum point.

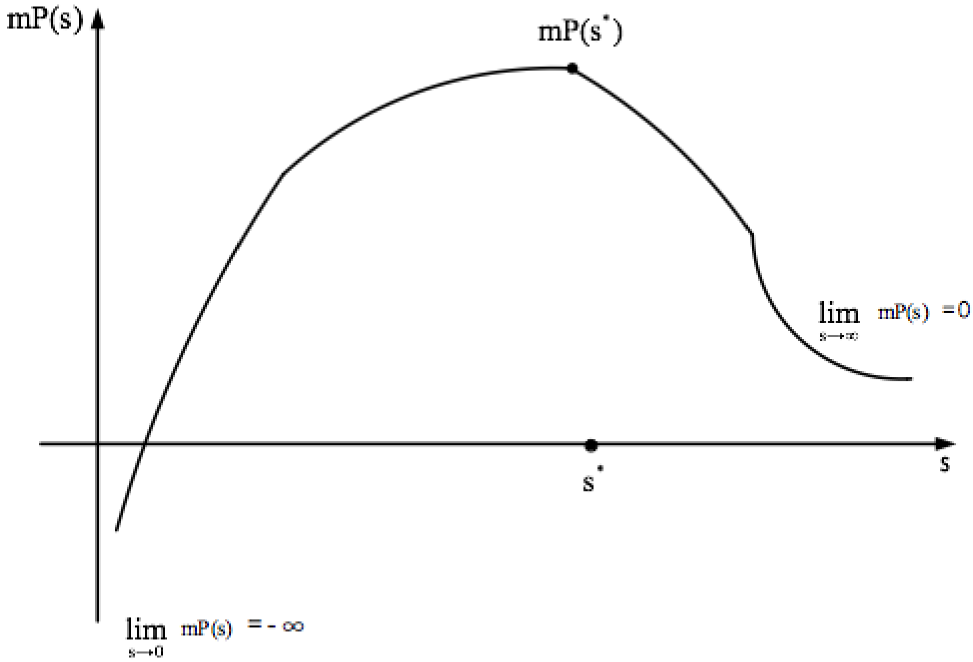

Figure 3. Graph of mP(s).

We summarize our results in the following theorem.

Theorem 1. We prove that $\frac{d}{ds} mP(s) = 0$ has a unique solution, denoted as s^*, and then s^* is the global maximum point.

From Equation (24), the optimal headway is derived as

$$h^* = \frac{s^*}{4jk}. \tag{30}$$

By Equation (7), the optimal service zone length is found as

$$L^* = \frac{(4j)^{a_2} q}{B_1 h^* s^* (2s^* + b)^{a_2}}. \tag{31}$$

Remark. In Chang and Schonfeld [8], Yang et al. [10] and Tung et al. [11], costs are linearly related by addition. They obtained a revised formulated optimal solution for the service route length and then they can obtain formulated solutions for service route width and headways. However, in Imam [9], profits are exponentially related by multiplication. Equation (27) only verified that $f(s) = 0$ has a unique positive solution and there seems to be no formulated solution for service zone width. Based on the above discussion, it can be claimed that the traffic models of Imam [9] and Chen and Julian [12] are more complicated than that of Chang and Schonfeld [8], Yang et al. [10], and Tung et al. [11].

6. Numerical example

Due to the absence of data for constant parameters for the traffic models in Imam [9], this paper referred to Chen and Julian [12] for a similar model to adopt the numerical example from Kocur and Hendrickson [17] with parameter values: $a_1 = 0.38$, $a_2 = -0.0081$, $a_3 = -0.0033$, $a_4 = -0.0014$, $a_5 = 0.0328$, $b = 0.2$, $c = 50$, $d = 3$, $F = 0.74$, $j = 0.05$, $k = 0.4$, $p = 3.59$, $q = 45$, $T = 60$, $v = 0.167$, and $W = 4$. Based on Equation (27), $f(s) = 0$ has a solution of $s^* = 0.9695$, by Equation (30), $h^* = 12.1187$, and by Equation (31), $L^* = 2.7859$ where the total profit is

$$mP\left(s^* = 0.9695,\ h^* = 12.1187,\ L^* = 2.7859\right) = 339.4508 \quad (32)$$

by Equation (9).

For completeness, the proposed derivation is compared with Chen and Julian [12] where $s^* = 0.731$, $h^* = 9.142$, and $L^* = 4.885$. Based on their derivations, the profit is evaluated using the corrected function shown as Equation (9) to find that

$$mP\left(s^* = 0.731, h^* = 9.142, L^* = 4.885\right) = 144.3016 \quad (33)$$

to indicate that the optimal solution proposed by Chen and Julian [12] is far from the optimal solution derived by this paper. If we compute that

, (34)

to show that our solution approach dramatically improves the maximum profit.

7. Conclusion

From the proposed derivations of two formulated relation among variables: (a) headways and the service route width of Equation (24), and (b) the service route length and the service route width of Equation (7), it is possible to rank parameters from most to least important. This allows researchers to allocate more of its budget in finalizing the values for those important parameters. If these critical parameters are too volatile, then it is possible to separate them into some small parts depending on cost (or profit) structure and time period during a day to obtain more stability. This subdivision will provide a good motivation for the future research of new traffic models.

Data Availability

Data in my numerical example is cited from Chen and Julian [12]. Data in numerical example of Chen and Julian [12] is cited from Kocur and Hendrickson [17]. Both Chen and Julian [12] and Kocur and Hendrickson [17] are published papers in Journal of Transportation Engineering.

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